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**DIFFUSIVE AND RADIATIVE EFFECTS ON VAPORIZATION  
TIMES OF DROPS IN FILM BOILING**

by Kenneth J. Baumeister  
Lewis Research Center  
Cleveland, Ohio

and

Glen J. Schoessow  
University of Florida  
Gainesville, Florida

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## DIFFUSIVE AND RADIATIVE EFFECTS ON VAPORIZATION

## TIMES OF DROPS IN FILM BOILING

by Kenneth J. Baumeister<sup>\*</sup> and Glen J. Schoessow<sup>†</sup>

## ABSTRACT

Diffusive and radiative effects are incorporated into an analysis for the vaporization time of drops in film boiling. The momentum, energy, and continuity equations are solved with some appropriate simplifications so as to obtain a simple closed form solution for the overall film boiling heat transfer coefficient. Next, a theoretical expression for the droplet vaporization time is developed and compared to the measured vaporization times of water droplets vaporizing into air, argon, nitrogen, and helium. The agreement between experiment and theory is good. Under the helium blanket, the diffusive evaporative component is significant in comparison to the film boiling component.

## INTRODUCTION

Studies of the vaporization time of liquid droplets in film boiling are basic to many technological problems. For example, current research in droplet film boiling on hot surfaces is concerned with such diverse problems as fuel vaporization on the

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<sup>\*</sup>Lewis Research Center, Cleveland, Ohio.

<sup>†</sup>University of Florida, Gainesville, Florida.

manifold of the spark-ignition engine or the combustion chamber of the diesel and gas turbine engine (ref. 1); cooling or quenching of heated surfaces (refs. 2 and 3); liquid droplet removal in mist sections of boilers (ref. 4); and in areas of safe fuel handling where accidental contact of fuel with a hot surface is a risk. The recent hazard study (ref. 5) of the problems associated with the accidental spillage of liquefied natural gas (LNG) on water also relates to the vaporization of large liquid drops in film boiling. References 1 to 15 contain a comprehensive current literature summary of the recent work in droplet film boiling as well as a discussion of the phenomenon.

In some of the above mentioned applications, drops vaporize in a saturated atmosphere of their own vapor, while in other applications the drop is exposed to an atmosphere in which diffusion can occur. With the exceptions of references 6 and 8, the vaporization times have been measured under conditions where the drop is exposed to air or other inert gases. Under such conditions, diffusion could significantly affect the total vaporization time. The purpose of the present paper is to obtain a closed-form solution for the effect of diffusion, as well as radiation, on the total vaporization time of drops in film boiling.

In analyzing the experimental data, two models of the boiling process have been considered. The first model, see figure 1(a), has no provisions for diffusion from the top of the

drop. In this model, the vapor is assumed to flow upward from beneath the drop and enshroud the upper surface. Under these conditions, diffusion from the upper surface can be neglected. Most of the analyses to date use this assumption with the exception of reference 10 by Gottfried and Bell. Bell (ref. 7) suggested the model shown in figure 1(b). In this model, the vapor from underneath the drop disperses radially outward away from the surface thereby leaving the upper surface exposed to an air or inert gas atmosphere. Under these conditions, molecular diffusion could occur. The fact that the droplet's upper regions are usually a few degrees below saturation temperature tends to support this model. Furthermore, Wachters (ref. 8) observed that a droplet in air evaporates faster than a droplet in a saturated atmosphere, thereby indicating that diffusive mass transfer exists.

In general, most of the data available are total vaporization times for pure liquid drops resting quiescently on a flat plate at high temperatures, exposed to an ambient air. Under these conditions, the diffusion contribution is generally small, less than 10 percent, compared to film boiling contribution. Consequently, it is difficult to discern which model is valid. Schoessow and Baumeister (ref. 6), however, performed an experiment where diffusion was a dominant factor. They noted that helium (see table 1) in association with water has a diffusion

coefficient which is a factor of four greater than an air-water combination. Figure 2 shows their data for the water droplet vaporization time in air, argon, nitrogen, and helium atmospheres. From this data and a dimensional analysis, Schoessow and Baumeister concluded that the second model (fig. 1(b)) is valid. To further support this conclusion, they presented a novel flow visualization technique which outlined the various domains associated with the second model.

Gottfried and Bell (ref. 14) included diffusion and radiation in an exact numerical solution for the two region model shown in figure 1(b). Their predicted vaporization times for small spherical drops in film boiling were in good agreement with their experimental results. On the other hand, Baumeister, Hamill, and Schoessow (ref. 11) neglected diffusion and obtained a single closed form solution based on model shown in figure 1(a). Their analysis applies to all drop sizes, small spheres to large liquid spills. Their predicted heat transfer coefficients and vaporization times were also in good agreement with the experimental results of reference 14. However, in light of recent substantiation of the two-region model, the simplified theory of reference 11 can only be used when diffusion is negligible.

The numerical results of Gottfried and Bell (ref. 14) and the theory of Baumeister et al. (ref. 11) compare favorably when diffusion is not a dominant mechanism. Unfortunately, the numer-

ical results of reference 14 cannot be compared to the theory to be presented in this paper because the numerical results of reference 14 applies only to specific drop sizes with an air atmosphere.

In the present paper, diffusive as well as radiative contributions are incorporated into an analysis for the heat transfer coefficient and vaporization time of drops in film boiling. The momentum and energy equations are solved with some appropriate simplifications to obtain a closed solution. A closed solution was desired for the convenience of the engineering designer. The theoretical results are compared to the measured droplet vaporization times of reference 6 for water droplets vaporizing into air, argon, nitrogen, and helium.

The analysis will treat radiation and diffusion as small perturbations to the well known solution of the Leidenfrost problem. The analysis is limited to two-component systems involving a pure single-component liquid with small solubilities for the gas and a single-component gas of high purity (with exception of air).

## METHOD OF ANALYSIS

The experimental measured vaporization time of a discrete liquid drop in film boiling can be determined by a direct integration of an energy balance on the drop:

$$-\lambda \rho_L \frac{dV}{dt} = h_T(V) A_B(V) \Delta T \quad (1)$$

where the total heat transfer coefficient,  $h_T$ , and the drop's bottom area  $A_B$  are dependent on the volume  $V$  of liquid that exists at any time  $t$ . Heat is assumed to reach the drop only through lower area of the drop. In equation (1), the total heat-transfer rate to the drop, represented by the right side of equation (1), is set equal to the mass evaporation rate times the latent heat of vaporization.

On the right-hand side of equation (1), the mechanism for energy transfer to the drop is assumed (see appendix A) to be conduction across the vapor film (in creeping laminar flow) and radiation to the bottom of the drop. On the left-hand side of equation (1), the mass evaporation is composed of diffusive vaporization from the top of the drop and film boiling from the

bottom.

The problem, of course, is to first relate  $A_B$  and  $h_T$  to the properties of the liquid and vapor, droplet volume, plate temperature, and the environmental conditions surrounding the drop. After this has been accomplished, equation (1) can be directly integrated to determine the vaporization time. First, the geometric parameters including  $A_B$  will be discussed. Secondly, an expression for the heat transfer coefficient will be presented. Finally, the integration of equation (1) will be discussed.

#### Geometric Parameter $A_B$

In this analysis, the drop is represented by a flat disk of radius  $r_0$  and height  $\ell$  as shown in figure 3. The uniform vapor gap  $\delta$ , shown in figure 3, is assumed to simplify the mathematics. For the cases where diffusive evaporation is small, this model gives realistic predictions of the heat transfer coefficient and vaporization time (ref. 11).

The relationship of  $A_B$ ,  $r_0$ , and  $\ell$  to droplet volume are given in table 2. The geometric parameters were determined in reference 11 from numerical solutions for the drop shape. From the numerical results, three analytical approximations were obtained for the various volume ranges listed in table 2.



## Heat Transfer Coefficient

An energy balance on the droplet can be written in the following form.

$$h_{TB} A_B \Delta T = \frac{k A_B \Delta T}{\delta} + h_{rad} A_B \Delta T = -\rho_v \lambda w(\delta) A_B + \frac{\lambda D_{AB} MP S_d}{RT_s r_o}$$

On the left hand side of the above equation, the total heat transferred,  $h_{TB} A_B \Delta T$ , is broken up into a conduction and a radiation term. The  $k A_B \Delta T / \delta$  term in the above equation assumes that only conduction occurs beneath the drop with the associated linear temperature profile. Convection heat transfer causes the temperature profile to deviate from the assumed linear value. Convection will be accounted for later by introducing the modified latent heat of vaporization. On the right hand side of the above equation, the first term represents heat removed from the drop's lower surface by evaporation, while the second term represents heat loss by diffusion from the upper surface. For this form of diffusive energy, the partial pressure of the liquid in the vapor environment is assumed small and an equivalent spherical shape has been assumed.

Dividing both sides of the above equation by  $A_B$  and  $\Delta T$  gives

$$h_T = \frac{k}{\delta} + h_{rad} = -\frac{\rho_v \lambda w(\delta)}{\Delta T} + \frac{\lambda D_{AB} MP S_d}{RT_s r_o \Delta T A_B}$$

The axial vapor velocity  $w(\delta)$  at the lower surface of the drop

can be found by a solution of the momentum and energy equations in the vapor flow field beneath the drop. Substituting, the expression for  $w(\delta)$  into the above equation leads after some algebraic manipulation to an expression for the total heat transfer coefficient  $h_T$ .

The detailed steps of the derivation for  $h_T$  are given in the appendix of this report. For ease of presentation, only the final expression for the total heat transfer coefficient is presented here in the body of the report.

The total heat transfer coefficient  $h_T$  can be expressed as:

$$h_T = \frac{h_o}{\left[ 1. - 0.25 N^* / V^{*1/4} - 0.75 h_{rad} / h_o \right]} \quad (2)-(A26)$$

((A-26) designates equation in appendix) where

$$N^* = \frac{1}{0.68} \frac{H}{S_c} \left( \frac{Pr^3}{GH^*} \right)^{1/4} \frac{A_d}{A_B} \quad (A25)$$

where the symbols are defined in the nomenclature.

In equation (2),  $h_o$  is the expression for the film boiling heat transfer coefficient when the diffusive contribution  $N^*$  (eq. (A25)) and the radiative contribution  $h_{rad}$  are zero. In this case,

$$h_o = 1.21 \left( \frac{V^*}{A^{*2}} \right)^{1/4} \left( \frac{k^3 \lambda^* (\rho_L - \rho_v) \rho_v g}{\Delta T \mu L} \right)^{1/4} \quad (3)-(A15)$$

The dimensionless volume and area parameter will be defined in

equations (9) and (10).

For convenience in the integration of equation (1), equation (3) is rearranged in the following form

$$h_o^* = \frac{h_o}{\left( \frac{k^3 \lambda^* (\rho_L - \rho_v) \rho_v g}{\Delta T \mu L} \right)^{1/4}} = 1.21 \left( \frac{V^*}{A^{*2}} \right)^{1/4} \quad (4)$$

where  $h_o^*$  is a dimensionless heat transfer coefficient. In a similar manner, the expression for the total heat transfer coefficient can be nondimensionalized by introducing equation (4) into equation (2). The expression for  $h_T$  becomes

$$h_T^* = \frac{h_o^*}{\left[ 1. - 0.25 \frac{N^*}{V^{*1/4}} - 0.75 \frac{h_{rad}^*}{h_o^*} \right]} \quad (5)$$

where

$$h_T^* = \frac{h_T}{\left( \frac{k^3 \lambda^* (\rho_L - \rho_v) \rho_v g}{\Delta T \mu L} \right)^{1/4}} = \frac{h_T}{\frac{k}{L} (G_r \text{Pr} H^*)^{1/4}} \quad (6)$$

and

$$h_{rad}^* = \frac{h_{rad}}{\left( \frac{k^3 \lambda^* (\rho_L - \rho_v) \rho_v g}{\Delta T \mu L} \right)^{1/4}} \quad (7)$$

In the next section, we will substitute the relationships for the area  $A$  and the total heat transfer coefficient  $h_T$  into equation (1) and integrate. This will lead to expressions

for the total vaporization time which can be compared to the experimental data in figure 2.

### Analysis of Vaporization Times

Defining

$$t^* = \frac{t}{\lambda \rho_L \left( \frac{\mu L^5}{k^3 \lambda^* g (\rho_L - \rho_v) \rho_v \Delta T^3} \right)^{1/4}} \quad (8)$$

$$V^* = \frac{V}{L^3} \quad (9)$$

$$A^* = \frac{A_B}{L^2} \quad (10)$$

$$L = \left[ \frac{\sigma g_c}{(\rho_L - \rho_v) g} \right]^{1/2} \quad (11)$$

and  $h_T^*$  as given in equation (4), equation (1) can be conveniently written in dimensionless form as:

$$-dV^* = h_T^* A^* dt^* \quad (12)$$

In this case,  $A^*$  represents the dimensionless bottom area of the flat disk model shown in figure 3. Integrating equation (12) gives:

$$t^* = \int_{V^*}^0 \frac{-dV^*}{h_T^* A^*} \quad (13)$$

The integration of equation (13) for  $V^*$  greater than 0.8

requires that the integration be broken up into two ranges since the form of  $h_T$  and  $A^*$  are different in each volume range. Consequently, for  $V^*$  greater than 0.8, the integration is from  $V^*$  to 0.8 and from 0.8 to zero. A similar approach is used for  $V^*$  greater than 155.

$$V^* \leq 0.8 \quad t^* = \int_0^{V^*} \frac{dV^*}{h_T^* A^*} \quad (14)$$

$$0.8 < V^* \leq 155 \quad t^* = \int_0^{0.8} \frac{dV^*}{h_T^* A^*} + \int_{0.8}^{V^*} \frac{dV^*}{h_T^* A^*} \quad (15)$$

$$V^* > 155 \quad t^* = \int_0^{0.8} \frac{dV^*}{h_T^* A^*} + \int_{0.8}^{155} \frac{dV^*}{h_T^* A^*} + \int_{155}^{V^*} \frac{dV^*}{h_T^* A^*} \quad (16)$$

where the appropriate form of the geometric parameter  $A^*$  and  $h_T$  is used in the evaluation of each of the above integrals. The above integrals are easily evaluated because all the integrals reduce to simple polynomials.

The result of these integrations is displayed in table 3. The formula relates the dimensionless time  $t^*$  to a pseudo liquid volume  $V^+$ . When diffusion and radiation are zero the pseudo volume becomes equal to the actual volume.

## COMPARISON OF EXPERIMENT TO THEORY

The experiment data of reference 6 are compared to the theoretical predictions in figure 4. In evaluating  $N^*$ , the diffusion coefficients in table 1 are used, the fluid and vapor properties are evaluated at the film temperature and the ratio of diffusive to film boiling area ( $A_d/A_B$ ) in  $N^*$  is taken as 1.35. Some diffusive evaporation has been assumed to take place from the sides of the drop, hence the thirty-five percent increase in the diffusive area as compared to the film boiling area. This average area ratio applies to drops in the dimensionless volume range of 0.8 to 155, which is the range of the experimental data.

Theory and experiment are in good agreement as seen in figure 4. The data fall within a standard deviation of 5 percent. In comparison, if the diffusive component of evaporation is neglected ( $D_{AB} = 0$ ), the helium data fall approximately 60 percent below the theoretical curve.

## LIMITATION OF THEORY

Various simplifications (linearizations) were used in deriving the expressions for the total heat transfer coefficient, see equations (A19), (A21), (A22), and (A23). These linearizations lead to a singularity in equation (2) in the limit when

$$\frac{0.25 N^*}{v^{*1/4}} + 0.75 \frac{h_{rad}}{h_o} \rightarrow 1$$

that is, the heat transfer coefficient becomes infinite.

For water drops vaporizing into a helium atmosphere, this singularity occurs at a water droplet volume of approximately 0.0001 cc. For large liquid drops, this singularity will have a negligible effect on the predicted total vaporization time; however, this will not be the case for very small liquid droplets. Consequently, it is suggested that the correlations in table III be used only in the range tested by experiment. From the present results, the last two equations in table 3 ( $V^* > 0.8$ ) can be safely used.

For very small liquid volumes ( $V^* < 0.8$ ), the numerical procedure of Gottfried, Lee, and Bell in reference 10 should be used for predicting the droplet vaporization time.

## CONCLUSIONS

A closed-form solution for the heat transfer coefficient and droplet vaporization time is in good agreement with data in which diffusion plays an important role.

## NOMENCLATURE

$A_B$	bottom area of drop
$A_d$	area of drop in which diffusion occurs
$A^*$	dimensionless area, defined by eq. (10)
$C_p$	specific heat at constant pressure of vapor

$D_{AB}$	diffusion coefficient (see table I)
$G_r$	defined by $\rho_v(\rho_L - \rho_v)gL^3/\mu^2$
$g$	coefficient of gravity
$g_c$	gravitational constant
$H$	defined by $\lambda/C_p \Delta T$
$H^*$	defined by $\lambda^*/C_p \Delta T$
$h$	heat transfer coefficient
$h_o$	film boiling heat transfer coefficient when radiation and diffusion are assumed zero
$h_{rad}$	radiative heat transfer coefficient $\epsilon_L \bar{\sigma}(T_w^4 - T_s^4)/(T_w - T_s)$
$h_T$	total heat transfer coefficient to bottom of the drop
$k$	thermal conductivity of vapor
$L$	characteristic length, equation (11)
$\ell$	thickness of flat disk drop model
$M$	molecular weight
$N_{DC}$	dimensionless diffusion parameter defined by equation (A24)
$N^*$	dimensionless diffusion parameter defined by equation (A25)
$P$	pressure
$P_o$	ambient pressure
$Pr$	Prandtl number $C_p \mu/k$
$P_s$	saturation pressure
$R$	gas constant
$r$	radial coordinate
$r_o$	effective droplet radius



$r_o^*$	dimensionless radius, $r_o/L$
$S_c$	modified Schmidt number, $\mu/D_{AB}(MP_s/RT_s)$
$T$	temperature
$T_f$	film temperature, equation (A1)
$T_p$	plate temperature
$T_s$	saturation temperature
$\Delta T$	temperature difference, $(T_p - T_s)$
$t$	time
$t^*$	dimensionless time, equation (8)
$u$	radial velocity
$V$	droplet volume
$V^*$	dimensionless droplet volume equation (9)
$V^+$	pseudo volume, see table 3
$w$	axial velocity
$w(\delta)$	axial velocity at bottom of drop
$Z$	axial coordinate
$\delta$	vapor gap thickness
$\delta_o$	vapor gap thickness when diffusion and radiation are not present
$\epsilon_L$	emissivity of liquid
$\lambda$	latent heat of vaporization

$\lambda^*$  modified latent heat of vaporization (ref. 13)

$$\begin{aligned} \frac{C_p \Delta T}{\lambda} \leq 2 \quad \lambda^* &= \lambda \left( 1 + \frac{7}{20} \frac{C_p \Delta T}{\lambda} \right)^{-3} \\ \frac{C_p \Delta T}{\lambda} > 2 \quad \lambda^* &= \lambda \left[ \frac{0.874 \ln \left( 1 + \frac{C_p \Delta T}{2\lambda} \right)}{\frac{C_p \Delta T}{2\lambda}} \right]^3 \end{aligned}$$

$\mu$  viscosity of vapor

$\rho_L$  liquid density

$\rho_v$  vapor density

$\sigma$  surface tension

$\bar{\sigma}$  Stefan-Boltzmann constant

## APPENDIX - HEAT TRANSFER COEFFICIENT

The heat transfer coefficient from the plate to the drop is obtained by solving the momentum and energy equations for flow and energy transfer in the vapor gap beneath the drop. The physical properties are assumed constant and evaluated at the film temperature:

$$T_f = \frac{T_p + T_s}{2} \quad (A1)$$

For creeping laminar flow with only heat conduction across the vapor gap, the governing differential equations are as follows:

Momentum:

$$0 = \frac{-g_c}{\rho_v} \frac{\partial P}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (A2)$$

$$0 = \frac{-g_c}{\rho_v} \frac{\partial P}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \quad (A3)$$

Continuity:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (A4)$$

Energy:

$$h = \frac{k}{\delta} \quad (A5)$$

Boundary conditions:

$$z = 0 \quad u = 0 \quad u = 0 \quad T = T_p \quad (A6)$$

$$z = \delta \quad u = 0 \quad w = w(\delta) \quad T = T_s \quad (A7)$$

$$r = 0 \quad u = 0 \quad (A8)$$

$$r = r_o \quad z = 0 \quad P = P_o \quad (A9)$$

Static force balance:

$$\int_0^{r_o} P(r, \delta) 2\pi r dr = \frac{g}{g_c} V(\rho_L - \rho_v) + \pi r_o^2 P_o \quad (A10)$$

Droplet energy balance:

$$h_{T-B} A_B \Delta T = \frac{k A_B \Delta T}{\delta} + h_{rad} A_B \Delta T = -\rho_v \lambda w(\delta) A_B + \frac{\lambda D_{AB}^{MP} A_d}{RT_s r_o} \quad (A11)$$

The set of equations presented above are similar to those given in reference 12 except that diffusion and radiation terms are added to the droplet energy balance. On the right-hand side of equation (A11), the first term represents heat removed from the drop by evaporation from the lower surface, while the second term represents heat loss by diffusion from the upper surface. For the form of diffusive energy loss, the partial pressure of the liquid in the vapor environment is assumed small and an equivalent spherical shape has been assumed.

The momentum equation along with the appropriate boundary conditions was solved in reference 12 to give

$$w(\delta) = \frac{2\pi}{3} \frac{(\rho_L - \rho_v) g \delta^3}{\mu L} \left( \frac{V^*}{A^{*2}} \right) \quad (A12)$$

where  $V^*$  and  $A^*$  are dimensionless volume and area of the bottom of the flat disk model, respectively. They are defined by equations (9) and (10) in the body of this report.

Equation (A12) could now be combined with the interface energy balance, equation (A11), to determine the gap thickness  $\delta$ . Next, the heat transfer coefficient can be found by combining the gap thickness  $\delta$  with equation (A5). However, for convenience, the vapor gap thickness and heat transfer coefficient will first be found for the case of pure film boiling.

For pure film boiling, no radiation or diffusing, the interface energy balance becomes

$$\frac{k \Delta T}{\delta_o} = -\rho_v \lambda w(\delta_o) \quad (A13)$$

where the subscript "0" has been added to signify that this is the gap thickness in the absence of diffusion or radiation. Substituting the expression for  $w(\delta)$  from equation (A12) into equation (A13) and solving for the vapor gap thickness gives

$$\delta_o = \left( \frac{3k \Delta T_{\mu} L}{2\pi\lambda(\rho_L - \rho_v)\rho_v g} \right)^{1/4} \left( \frac{A}{V^*} \right)^{1/4} \quad (A14)$$

From the definition of the heat transfer coefficient in equation (A5),

$$h_o = \frac{k}{\delta_o} = 1.21 \left( \frac{k^3 \lambda^* (\rho_L - \rho_v) \rho_v g}{\Delta T_{\mu} L} \right)^{1/4} \left( \frac{V^*}{A} \right)^{1/4} \quad (A15)$$

where the latent heat of vaporization  $\lambda$  has been replaced by the modified latent heat of vaporization  $\lambda^*$ . The modified latent heat of vaporization accounts for the convective terms in the energy equation which were neglected in equation (A5). The

function form of  $\lambda^*$  was derived in references 12 and 13 and is given in the list of symbols.

Now, substituting equations (A12) and (A13) into equation (A11) gives

$$\delta^4 = \delta_o^4 \left\{ 1 - \frac{\delta}{k} \left[ \frac{\lambda_{AB}^{MP} s}{RT_s r_o \Delta T} \left( \frac{A_d}{A_B} \right) - h_{rad} \right] \right\} \quad (A16)$$

or multiplying the diffusive and radiative terms by the identity  $\delta_o/\delta_o$  gives

$$\delta^4 = \delta_o^4 \left[ 1 - \left( N_{DC} - \frac{h_{rad}}{h_o} \right) \frac{\delta}{\delta_o} \right] \quad (A17)$$

where

$$N_{DC} = \frac{A_d}{A_B} \frac{\lambda_{AB}^{MP} s}{RT_s r_o h_o \Delta T} = \frac{H}{S_c} \left( \frac{Pr^3}{G_r H^*} \right)^{1/4} \frac{1}{h_o^* r_o^*} \frac{A_d}{A_B} \quad (A18)$$

The definitions of  $h_o^*$  and  $r_o^*$  are given in table 2. This dimensionless group is a measure of the importance of diffusion from the top of the drop to conduction below.

Equation (A16) is the biquadratic equation for which solutions exist (ref. 16). Unfortunately, the more general solution does not lead to a closed-form solution for the total vaporization time. However, for the range of  $N_{DC}$  and  $h_{rad}$  associated with typical experiments, a simple iterative solution can be found by assuming  $\delta \approx \delta_o$  on the right-hand side of equation (A17).

This gives

$$\delta = \delta_o [1 - (N_{DC} - h_{rad}/h_o)]^{1/4} \quad (A19)$$

The total heat transfer coefficient to the drop can now be found by substituting  $\delta$  into the left-hand side of equation (A11).

$$h_T = \frac{h_o}{[1 - (N_{DC} - h_{rad}/h_o)]^{1/4}} + h_{rad} \quad (A20)$$

This form of the total heat transfer, although relatively simple, will still not lead to a closed-form solution for the total vaporization time. Therefore, equation (A20) is now written in the form of a binomial expansion in which only the first order terms are kept. The details are as follows.

$$h_T \approx \frac{h_o}{1 - \frac{1}{4} (N_{DC} - h_{rad}/h_o)} + h_{rad} \quad (A21)$$

or

$$h_T = h_o \left[ 1 + \frac{1}{4} \left( N_{DC} - \frac{h_{rad}}{h_o} \right) \right] + h_{rad} = h_o \left[ 1 + \frac{1}{4} \left( N_{DC} + \frac{3h_{rad}}{h_o} \right) \right] \quad (A22)$$

Finally, inverting gives

$$h_T = \frac{h_o}{1 - \frac{1}{4} (N_{DC} + 3h_{rad}/h_o)} \quad (A23)$$

The diffusion parameter  $N_{DC}$  is a function of the dimensionless volume  $V^*$  which is embedded in the terms  $h_o^*$  and  $r_o^*$  in equation (A18). Substituting in the value of  $h_o^* r_o^*$  from table 2 into equation (A17) gives

$$N_{DC} = \frac{1}{0.68} \frac{H}{S_c} \left( \frac{Pr^3}{G_r H^*} \right)^{1/4} \left( \frac{A_d}{A_B} \right) \frac{1}{V^{*1/4}} \quad (A24)$$

Defining

$$N^* = \frac{1}{0.68} \frac{H}{S_c} \left( \frac{Pr^3}{G_r H^*} \right)^{1/4} \left( \frac{A_d}{A_B} \right) \quad (A25)$$

The heat transfer coefficient to the drop becomes

$$h_T = \frac{h_o}{\left[ 1. - 0.25 N^* / V^{*1/4} - 0.75 h_{rad}/h_o \right]} \quad (A26)$$

In this case,  $N^*$  is independent of drop volume. This form of the total heat transfer coefficient leads to a closed solution for the total vaporization time.



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TABLE 1. - DIFFUSION COEFFICIENTS (373 K  
AND ATMOSPHERIC PRESSURE)

Liquid-gas combination	$D_{AB}$ , cm <sup>2</sup> /sec	Refer- ence	Comments
Water-air	0.353	16	---
Water-argon	0.41	17	(a)
Water-helium	1.22	17	(b)
Water-nitrogen	0.40	17	(b)

<sup>a</sup> $D_{AB}$  calculated by Slattery-Bird equation corrected for water (eq. (11-27)) as given in ref. 17.




<sup>b</sup>Experimental data given in table 11-4 of ref. 17 was linearly extrapolated to 373 K.

TABLE 2. - RADIUS AND CONDUCTIVE HEAT TRANSFER COEFFICIENT FOR DROPLET FILM BOILING

Dimensionless volume, $V^* = \frac{V}{L^3}$	Drop shape	$A^* = \frac{A_B}{L^2}$	Dimensionless radius, $r_o^* = \frac{r}{L}$	Dimensionless heat transfer coefficient, $h_o^* = 1.21 \left( \frac{V^*}{A^*} \right)^{1/4}$	$r_o^* h_o^*$
$V^* < 0.8$	Small spheroid 	$A^* = 1.81 V^{*2/3}{}^{\dagger}$	$r_o^* = \left( \frac{3V^*}{4\pi} \right)^{1/3}$	$h_o^* = 1.1 V^{*-1/12}$	$0.68 V^{*1/4}$
$0.8 < V^* < 155$	Large drop 	$A^* = 1.25 V^{*5/6}$	$r_o^* = \left( \frac{1.25 V^{*5/6}}{\pi} \right)^{1/2}$	$h_o^* = 1.075 V^{*-1/6}$	$0.68 V^{*1/4}$
$V^* > 155$	Extended drop { constant thickness } 	$A^* = 0.54 V^*$	$r_o^* = \left( \frac{0.54 V^*}{\pi} \right)^{1/2}$	$h_o^* = 1.64 V^{*-1/4}$	$0.68 V^{*1/4}$

${}^{\dagger}A^*$  was modified empirically in reference 11 to account for spherical shape.

TABLE 3. - THEORETICAL EXPRESSIONS FOR DROPLET VAPORIZATION TIME

Dimensionless volume, $v^* = \frac{V}{L^3}$	Drop shape	$v^+$	$t^*$
$v^* < 0.8$	Small spheroid 	$v^+ = \left( v^{*5/12} - 0.624 N^* v^{*1/6} - 0.142 h_{rad}^* v^{*1/2} \right)^{12/5}$	$t^* = 1.205 v^{*5/12}$
$0.8 < v^* < 155$	Large drop 	$v^+ = \left( v^{*1/3} - 1.345 N^* v^{*1/12} + 0.995 N^* - 0.465 h_{rad}^* v^{*1/2} + 0.345 h_{rad}^* \right)^3$	$t^* = 2.23 v^{*1/3} - 0.97$
$v^* > 155$	Extended drop {Constant thickness} 	$v^+ = \left[ v^{*1/4} + 0.0625 N^* \ln\left(\frac{v^*}{155}\right) + 2.29 h_{rad}^* v^{*1/2} - 0.524 N^* - 32.3 h_r^* \right]^4$	$t^* = 4.52 v^{*1/4} - 5$

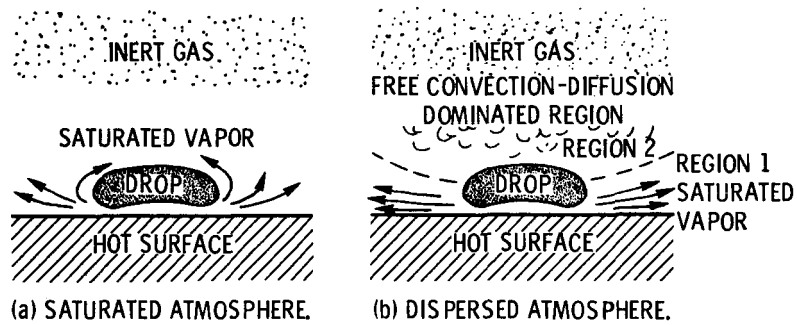


Figure 1. - Velocity flow models for drops in film boiling.

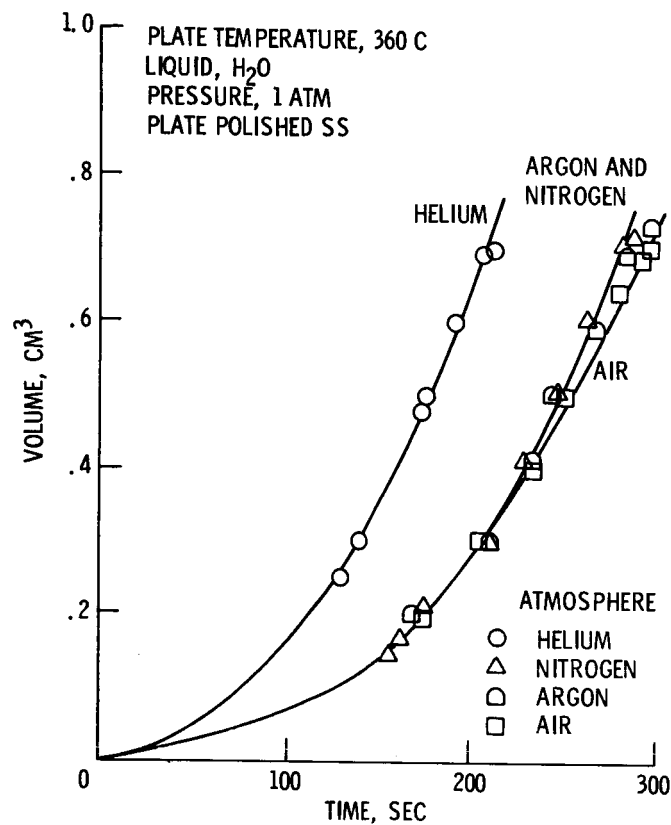


Figure 2. - Total vaporization data for various water and inert gas combinations.

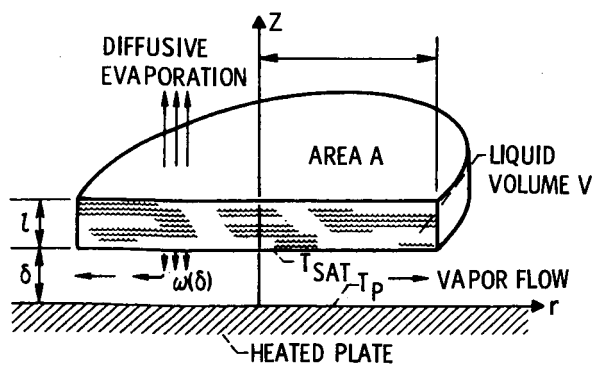


Figure 3. - Schematic model for evaporation of flat disk.

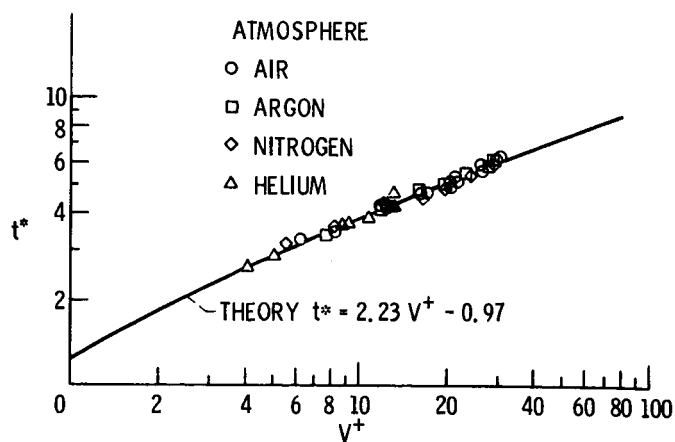


Figure 4. - Total vaporization time correlation for water droplets (data from ref. 6).